(9) $V = M_{n\times n}(\mathbb{F})$

$\dim V = n^2 \cdot \dim(\mathbb{F}(\mathbb{F})) = n^2 \cdot 1 = n^2$

$|AA^T = A^TA = I|$ ORTHOGONAL

Q: Let $V(\mathbb{R}) = \{\langle a_n\rangle : a_n \in \mathbb{R}\}$. Then,

~~(A) dim V = finite~~ ; ~~(B) dim V = uncountable~~

(C) dim V = countable ✓ ; ~~(D) dim V = countably finite~~ FINITE

$\langle a_n\rangle = \{a_1, a_2, \ldots\}$ $f : N \to \mathbb{R}$; $B = \{e_1, e_2, \ldots\}$

$|B| = \text{card}(B) = \infty$ < COUNTABLE ($\mathbb{N}, \mathbb{Z}, \mathbb{Q}$) / INFINITELY COUNT. ($\mathbb{R}, \mathbb{Q}^c$) → INFINITE.

§ COUTABLY INFINITE → alpha Not $= \aleph_0$

UNCOUNTABLY INFINITE = continuum = c

Quest: Let $V(\mathbb{R}) = M_{n\times n}(\mathbb{R})$ and

$W_1 = \{A \in V : A^T = A\}$ (SYMM.)

$W_2 = \{A \in V : A^T = -A\}$ (SKEW SYMM.)

Then, W.O.T.F.

(A) $\dim(W_1) = \frac{n(n+1)}{2}$ ✓ , (B) $\dim(W_1) = \frac{n(n-1)}{2}$

(C) $\dim W_2 = \frac{n(n+1)}{2}$ (D) $\dim(W_2) = \frac{n(n-1)}{2}$ ✓

SYMM: $\begin{bmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & & a_{nn} \end{bmatrix}$ diag. entries $= \boxed{n}$ elts.

off-diag. entries $= \frac{n^2-n}{2}$ elts.

[$a_{12} \to a_{21}$ (generates)]

$\therefore n^2 - \frac{(n^2-n)}{2} + n = \frac{2n^2 - n^2 - n + 2n}{2} = \frac{n^2+}{2}$

$= \frac{n(n+1)}{2}$

SKEW-SYMM.

$\begin{bmatrix} 0 & & \\ & 0 & \\ & & \ddots \\ & & & 0 \end{bmatrix}$ $0 + \frac{n^2-n}{2} = \frac{n(n-1)}{2}$

hermitian m

$|AA^T = A^TA = I|$ ORTHOGONAL

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⑨ $V = M_{n\times n}(F)$

$\dim V = n^2 \cdot \dim(F(F)) = n^2 \cdot 1 = n^2$

Q: Let $V(\mathbb{R}) = \{\langle a_n\rangle : a_n \in \mathbb{R}\}$. Then,

× (A) $\dim V =$ finite ; × (B) $\dim V =$ uncountable

✓ (C) $\dim V =$ countable (D) × $\dim V =$ countably finite / a FINITE

$\langle a_n\rangle = \{a_1, a_2, \ldots\}$ $f : N \to \mathbb{R}$; $B = \{e_1, e_2, \ldots\}$

$|B| = \text{card}(B) = \infty$ < COUNTABLE $(\mathbb{N}, \mathbb{Z}, \mathbb{Q})$ / INFINITELY COUNT. $(\mathbb{R}, \mathbb{Q}^c) \to$ INFINITE.

{ COUTABLY INFINITE $\longrightarrow$ alpha Not $= \aleph_0$ }

{ UNCOUNTABLY INFINITE $=$ continuum $= c$ }

Quest: Let $V(\mathbb{R}) = M_{n\times n}(\mathbb{R})$ and

$W_1 = \{A \in V : A^T = A\}$ (SYMM.)

$W_2 = \{A \in V : A^T = -A\}$ (SKEW SYMM.)

Then, W.O.T.F.

✓ (A) $\dim(W_1) = \frac{n(n+1)}{2}$, (B) $\dim(W_1) = \frac{n(n-1)}{2}$

(C) $\dim W_2 = \frac{n(n+1)}{2}$ ✓ (D) $\dim(W_2) = \frac{n(n-1)}{2}$

SYMM:

$$\begin{bmatrix} a_{11} & & & \\ & a_{22} & & \\ & & \ddots & \\ & & & a_{nn} \end{bmatrix}$$

diag. entries $= \boxed{n}$ elts.

off-diag. entries $= \frac{n^2 - n}{2}$ elts.

$[a_{12} \to a_{21}$ (generates)$]$

$\therefore \quad n^2 - \frac{(n^2 - n)}{2} + n = \frac{2n^2 - n^2 - n + 2n}{2} = \frac{n^2 + n}{2}$

$= \frac{n(n+1)}{2}$

SKEW-SYMM.

$$\begin{bmatrix} 0 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}$$

$0 + \frac{n^2 - n}{2} = \frac{n(n-1)}{2}$

Quest: Let $V = M_{4\times4}(\mathbb{C})$ and $W = \{M \in V ; M^T = \bar{M}\}$ be a subsp. of V. Then $\dim(W) =$ (hermitian matrices)

(A) 6 (B) 12 (C) 16 (D) 20